Constructing Approximate Solutions in Regularization Methods for Linear Discrete Ill-Posed Problems

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Abstract

This talk explores GMRES methods as regularization methods for a stability problem of linear discrete ill-posed problems. Regularization with GMRES must be implemented in two stages, which are designed to generate an approximate solution of a linear system through the use of GMRES, and to determine the most appropriate solution by using a constraint. We will propose new methods mainly for the convenience of users.

In the first stage, an augmented GMRES method with user-supplied spaces was suggested. Not to worried about which space to augment we propose a selecting condition that automatically determines the adequate one from several candidates for generating an aproximate solution.

In the second stage, a simplified Tikhonov value as a constraint to determine the best approximate solution is explored. The method we have developed is a modification of Tikhonov regularization. The classical iterative method often uses a residual norm as an index, but the Tikhonov regularization method uses a solution norm in addition to this. The proposed value consists of several values naturally appearing in GMRES iteration, so that the need for extra computation is minimized. The simplified Tikhonov value makes it possible for GMRES to stop iteration adaptively.

Numerical experiments have been tabulated to underline the properties of linear discrete ill-posed problems and the effectiveness of our proposed method.